EE3123 Tutorial 6 (Solution)

Additional: Three-phase transformer, Transient stability

Name: **Student No.:**

Q1

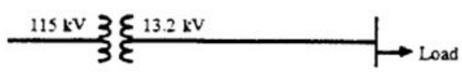
A three phase transformer rated 5 MVA, 115/13.2 kV has an old per-phase series impedance of (0.007+j0.075) per unit. The transformer is connected to a short distribution line which can be represented by a series impedance per phase of (0.02+j0.1) per unit on a new base of 10 MVA, 13.2 kV. The line supplies a balanced three phase load rated 4 MVA, 13.2 kV, with a lagging power factor 0.85.

a) Draw an equivalent circuit of the system indicating all impedances in per unit. Choose 10 MVA, 13.2 kVA as the base at the load.

b) With the voltage at primary side of the transformer held constant at 115 kV, the load at the receiving end of the line is disconnected. Find the voltage regulation at the load (output voltage changed after disconnection).

Solution

(*a*) Base voltages are shown on the single-line diagram.



$$\text{Transformer } Z = \frac{10}{5}(0.007 + j0.075) = 0.014 + j0.150 \text{ per unit}$$

$$V_S = 1.0 \text{ per unit}$$

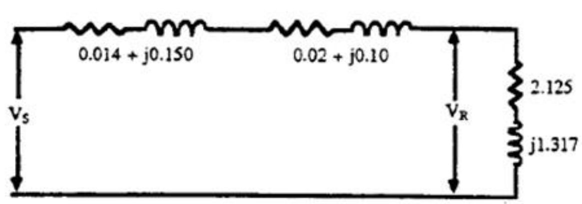
$$\text{Line } Z = 0.02 + j0.10 \text{ per unit}$$

$$\text{Load } |Z| = \frac{(13.2)^2 \times 1000}{4000 / 3} = 130.68\ \Omega$$

$$\text{Base } Z \text{ at load} = \frac{(13.2)^2}{10/3} = 52.26\ \Omega$$

$$\text{Load } Z = \frac{130.68}{52.26}\angle \cos^{-1} 0.85 = 2.50\angle 31.8^\circ$$

$$= 2.125 + j1.317 \text{ per unit}$$



(values are in per unit)

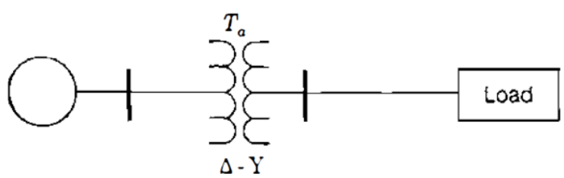
(b) Voltage regulation calculations

$$I = \frac{1.0}{0.014 + 0.02 + 2.125 + j(0.150 + 0.10 + 1.317)} = \frac{1.0}{2.668\angle 35.97^\circ}$$
$$= 0.375\angle -35.97^\circ \text{ per unit}$$
$$V_{R,FL} = 0.375\angle -35.97^\circ \times 2.5\angle 31.8^\circ = 0.937\angle -4.17^\circ \text{ per unit}$$
$$V_{R,NL} = V_S = 1.0$$
$$\text{V.R.} = \frac{1 - 0.937}{0.937} \times 100 = 6.72\ \%$$

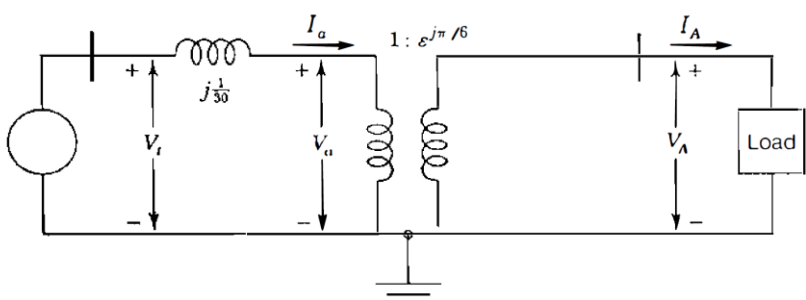
Q2

The below figure shows a three phase generator rated 300 MVA, 23 kV supplying a system load of 240 MVA, 0.9 power factor lagging at 230 kV through a 330 MVA 23-Δ/230-Y kV step-up transformer of leakage reactance 0.333 p.u. Neglecting magnetizing current and choosing base values at the load of 100 MVA and 230 kV.

1) Find load I_A, I_B, and I_C supplied to the load in per unit with load voltage V_A as the reference.
2) Specifying the proper base for the generator circuit, determine I_a, I_b, and I_c from the generator and its terminal voltage.



Solution



The current supplied to the load is

$$\frac{240{,}000}{\sqrt{3} \times 230} = 602.45 \text{ A}$$

The *base* current at the load is

$$\frac{100{,}000}{\sqrt{3} \times 230} = 251.02 \text{ A}$$

The power-factor angle of the load current is $\theta = \cos^{-1} 0.9 = 25.84^\circ$ lag

Hence, with $V_A = 1.0\angle 0°$ as reference, the line currents

$$I_A = \frac{602.45}{251.02}\angle -25.84° = 2.40\angle -25.84° \text{ per unit}$$

$$I_B = 2.40\angle -25.84° - 120° = 2.40\angle -145.84° \text{ per unit}$$

$$I_C = 2.40\angle -25.84° + 120° = 2.40\angle 94.16° \text{ per unit}$$

Low-voltage side currents further lag by 30°, and so in per unit

$$I_a = 2.40\angle -55.84° \qquad I_b = 2.40\angle\ 175.84° \qquad I_c = 2.40\angle 64.16°$$

the terminal voltage of the generator is

$$V_t = V_A\angle -30° + jXI_a$$

$$= 1.0\angle -30° + \frac{j}{30} \times 2.40\angle -55.84°$$

$$= 0.9322 - j0.4551 = 1.0374\angle -26.02° \text{ per unit}$$

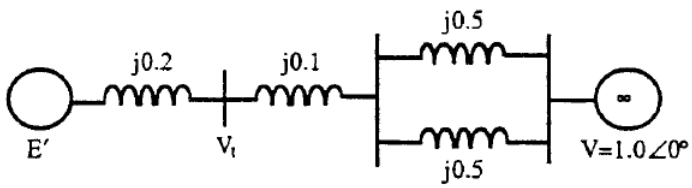
The base generator voltage is 23 kV, which means that the terminal voltage of the generator is 23 × 1.0374 = 23.86 kV. The real power supplied by the generator is

$$\text{Re}\{V_t I_a^*\} = 1.0374 \times 2.4 \cos(-26.02° + 55.84°) = 2.160 \text{ per unit}$$

which corresponds to 216 MW absorbed by the load since there are no I^2R losses.

Q3

A power system is shown as below, where the impedance of each of the paraellel transmission lines is j0.5 and the deliverd power is 0.8 per unit when both the terminal voltage Vt of the generator output and the voltage of the infinite bus are 1.0 per unit. Determine the power-angle equation for the system during the specified operating conditions.



Solution

X between V_t and V is

$$j0.1 + \frac{j0.5}{2} = j0.35 \text{ per unit}$$

If $V_t = 1.0\angle\delta_t$,

$$\frac{1.0 \times 1.0}{j0.35}\sin\delta_t = 0.8, \qquad \delta_t = 16.26°$$

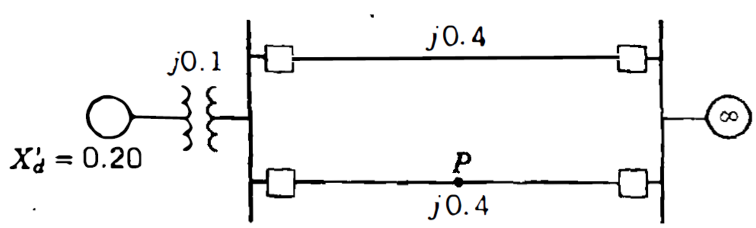
$$I = \frac{1.0\angle 16.26° - 1.0\angle 0°}{0.35\angle 90°} = \frac{0.96 + j0.28 - 1.0}{j0.35}$$
$$= 0.8 + j0.1143 = 0.8081\angle 8.13°$$
$$E' = 1.0\angle 16.26° + 0.808\angle 8.13° \times 0.2\angle 90°$$
$$= 0.96 + j0.28 - 0.023 + j0.16 = 1.0352\angle 25.15°$$
$$P_e = \frac{1.0352 \times 1.0}{0.35 + 0.20} \sin\delta = 1.882 \sin\delta$$

Note: δ is different from δ_t

Q4

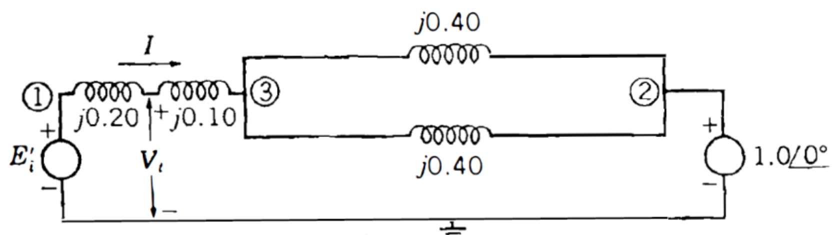
The single-line diagram, as shown below, through parallel transmission lines to a large metropolitan system considered as an infinite bus. The machine is delivering 1.0 per unit power and both the terminal voltage *Vt* and the infinite-bus voltage are 1.0 per unit. Numbers on the diagram indicate the values of the reactances on a common system base. The transient reactance of the generator is 0.2 per unit.

1) Prefault: Determine the power angle equation for the system.
2) During fault: A grounded fault occurs at point P, determine the power angle equation for the system and the corresponding swing equation. H=5 MJ/MVA.
3) Postfault: The fault is cleared by simultaneous opening of the circuit breakers at each end of the affected line. Determine the critical clearing angle.



Solution

1)



(a)

reactance between the terminal voltage and the infinite bus is

$$X = 0.10 + \frac{0.4}{2} = 0.3 \text{ per unit}$$

and therefore the 1.0 per-unit power output of the generator is determined by

$$\frac{|V_t||V|}{X}\sin \delta_t = \frac{(1.0)(1.0)}{0.3}\sin \delta_t = 1.0$$

where V is the voltage of the infinite bus and δ_t is the angle of the terminal voltage relative to the infinite bus. Solving for δ_t, we obtain

$$\delta_t = \sin^{-1} 0.3 = 17.458^\circ$$

so that the terminal voltage is

$$V_t = 1.0\angle 17.458^\circ = 0.954 + j0.300 \text{ per unit}$$

The output current from the generator is now calculated as

$$I = \frac{1.0\angle \delta_2 - 1.0\angle 0^\circ}{j0.3}$$

$$= 1.0 + j0.1535 = 1.012\angle 8.729^\circ \text{ per unit}$$

and the transient internal voltage is then found to be

$$E_i' = (0.954 + j0.30) + j(0.2)(1.0 + j0.1535)$$

$$= 0.923 - j0.5 = 1.050\angle 28.44^\circ \text{ per unit}$$

The power-angle equation relating the transient internal voltage E_i' and the infinite-bus voltage V is determined by the *total* series reactance

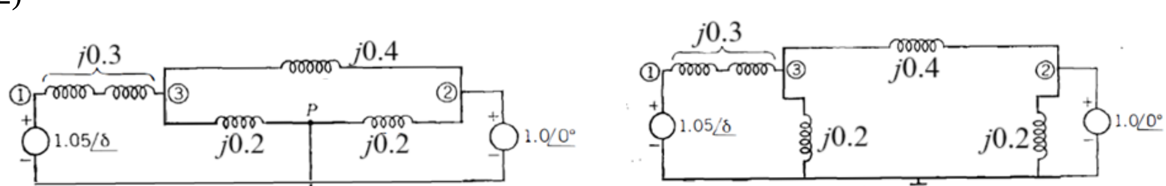
$$X = 0.2 + 0.1 + \frac{0.4}{2} = 0.5 \text{ per unit}$$

Hence, the desired equation is

$$P_e = \frac{(1.050)(1.0)}{0.5}\sin \delta = 2.10 \sin \delta \text{ per unit}$$

where δ is the machine rotor angle with respect to the infinite bus.

2)



(b) (c)

By using the Thevenin equivalent method, the equivalent impedance can be calculated, with unchanged generator voltage,

The power-angle equation with the fault on the system is therefore

$$P_e = 0.808 \sin \delta \text{ per unit}$$

and the corresponding swing equation is

$$\frac{5}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin\delta \text{ per unit}$$

3)

The net transfer impedance is j(0.2+0.1+0.4)

Therefore, the postfault power-angle equation is

$$P_e = (1.05)(1.0)(1.429)\sin\delta = 1.500 \sin\delta$$

The power-angle equations obtained in the previous examples are

Before the fault: $P_{max} \sin\delta = 2.100 \sin\delta$

During the fault: $P_{max} \sin\delta = 0.808 \sin\delta$

After the fault: $P_{max} \sin\delta = 1.500 \sin\delta$

$$\delta_{max} = 180^\circ - \sin^{-1}\left[\frac{1.000}{1.500}\right] = 138.190^\circ = 2.412 \text{ rad}$$

Based on Equal-area criterion

$$\int_{\delta_0}^{\delta_c}(P_m - P_e)\,d\delta = \int_{\delta_c}^{\delta_x}(P_e - P_m)\,d\delta$$

$$\cos\delta_{cr} = \frac{\left(\frac{1.0}{2.10}\right)(2.412 - 0.496) + 0.714\cos(138.19^\circ) - 0.385\cos(28.44^\circ)}{0.714 - 0.385}$$

$$= 0.127$$

Hence,

$$\delta_{cr} = 82.726^\circ$$